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Final Exam (80 points)
Multiple Choice: Select the best answer for each of the following questions. Write your answer as an English letter to the left of each problem.

1. Three point charges of $-2.00 \mu \mathrm{C},+4.00 \mu \mathrm{C}$, and $+6.00 \mu \mathrm{C}$ are placed along the $x$-axis as shown in the figure. What is the electrical potential at point $P$ (relative to infinity) due to these charges?

(a) -307 kV
(b) +307 kV
(c) -154 kV
(d) +154 kV
(e) 0 kV
$\left(2^{\text {pts }}\right) \quad$ 2. A charged capacitor stores energy $U$. Without connecting this capacitor to anything, dielectric having dielectric constant $\kappa$ is now inserted between the plates of the capacitor, completely filling the space between them. How much energy does the capacitor now store?
(a) $2 \kappa U$
(b) $\kappa U$
(c) $U$
(d) $U / \kappa$
(e) $U /(2 \kappa)$
$\left(2^{\text {pts }}\right)$ 3. Four point charges, each of the same magnitude, but with varying signs, are arranged at the corners of a square as shown. Which of the arrows labelled $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and $\mathbf{D}$ gives the correct direction of the net force that acts on the charge at the upper right corner?

(a) $\mathbf{A}$
(b) $\mathbf{B}$
(c) $\mathbf{C}$
(d) $\mathbf{D}$
(e) The net force is zero.
(2 $\left.2^{\text {pts }}\right)$ 4. Which surface, A or B, has the largest electrical flux? A is the vertical rectangular surface and $B$ is the broad sloped surface.

(a) A
(b) B
(c) The electrical fluxes are equal.
(d) More information is needed.
$\left(2^{\mathrm{pts}}\right)$
2. Which resistor dissipates more power?

(a) $1 \Omega$
(b) $9 \Omega$
(c) They dissipate the same power.
(d) No power is dissipated.
$\left(2^{\text {pts }}\right)$
3. $\mathrm{A}+1.0 \mu \mathrm{C}$ point charge is moved from point $\mathbf{A}$ to $\mathbf{B}$ in the uniform electric field as shown. What happens to potential energy of the point charge as a result of this move?

(a) It increases by $6.0^{-6} \mathrm{~J}$
(b) It decreases by $6.0^{-6} \mathrm{~J}$
(c) It decreases by $9.0^{-6} \mathrm{~J}$
(d) It increases by $10.8^{-6} \mathrm{~J}$
(e) It decreases by $10.8^{-6} \mathrm{~J}$
$\left(2^{\mathrm{pts}}\right)$
4. A solid nonconducting sphere of radius $R$ carries a uniform charge density throughout its volume. At a radial distance $r_{1}=R / 4$ from the centre, the electric field has a magnitude $E_{0}$. What is the magnitude of the electric field at a radial distance $r_{2}=2 R$ ?
(a) $E_{0} / 4$
(b) zero
(c) $E_{0} / 2$
(d) $E_{0}$
(e) $2 E_{0}$
5. The figure shows four different sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of the current flow are as indicated. For which, if any, configuration will the magnetic field at the centre of the square formed by the wires be equal to zero?

(a) $\mathbf{A}$
(b) $\mathbf{B}$
(c) $\mathbf{C}$
(d) $\mathbf{D}$
(e) The field is not equal to zero in any of these cases.
$\left(2^{\mathrm{pts}}\right)$
6. Three very long, straight, parallel wires each carry currents of 4.00 A directed out of the page as shown in the figure. The wire pass through the vertices of a right isosceles triangle of side 2.00 cm . What is the magnitude of the magnetic field at point $P$ at the midpoint of the hypotenuse of the triangle?

(a) $1.77 \times 10^{-6} \mathrm{~T}$
(b) $4.42 \times 10^{-6} \mathrm{~T}$
(c) $1.77 \times 10^{-5} \mathrm{~T}$
(d) $5.66 \times 10^{-5} \mathrm{~T}$
(e) $1.26 \times 10^{-4} \mathrm{~T}$
$\left(2^{\text {pts }}\right)$ 10. In the figure, two parallel wires carry currents of magnitude $I$ in opposite directions. A rectangular loop is midway between the wires. The current $I$ is decreasing with time. The induced current through resistor $R$ is:

(a) from $a$ to $b$.
(b) from $b$ to $a$
(c) There is no induced current through the resistor.


Free Response: Write out complete answers to the following questions. Show your work since it allows us to be generous with partial credit.
(10 ${ }^{\mathrm{pts}}$ ) 11. a) Two identical positively charged conducting spheres are fixed in space. The spheres are 32.6 cm apart (centre to centre) and repel each other with an electrostatic force $F_{1}=0.0615 \mathrm{~N}$. Then, a thin conducting wire connects the spheres, redistributing the charge on each sphere. When the wire is removed the spheres still repel but with a force of $F_{2}=0.115 \mathrm{~N}$. Use this information to find the initial charge on each sphere. Assume that initially $q_{1}<q_{2}$. ( 6 marks)
b) Initially, a conducting sphere of radius $R$ has charge $Q$. Nearby, there is a neutral conducting of radius $R / 3$. The two spheres are connected by a thin conducting wire. After connecting the wire, what is the charge on the smaller sphere? Express you answer in terms of $Q$.
(10 $\left.0^{\text {pts }}\right)$ 12. a) You are in the lab and need a 90 pF capacitor to properly set the time constant of an $R C$ circuit. Unfortunately, the lab is not equipped with 90 pF capacitors. Your TA gives you one $80 \mathrm{pF} \pm 10 \%$ capacitor, one $80 \mathrm{pf} \pm 15 \%$ capacitor, and one $50 \mathrm{pF} \pm 20 \%$ capacitor. Your TA tells you to combine these capacitors make the necessary 90 pF . How should you combine these capacitors? Draw a circuit diagram and label which capacitors are 80 pF and which is 50 pF . (5 marks)
b) What is the uncertainty in the capacitance of the 90 pF circuit that you constructed in part a)? Give your answer in pF. (5 marks)
( $\left.10^{\text {pts }}\right)$ 13. a) Find the currents in the circuit shown in figure a). Are they shown in the correct direction? (Do not change the directions of the currents.) (5 marks)
b) When the current in the portion of the circuit shown in figure b) is 2.00 A and increases at a rate of $0.500 \mathrm{~A} / \mathrm{s}$, the measured voltage is $\Delta V_{a b}=9.00 \mathrm{~V}$. When the current is 2.00 A and decreases at the a rate of $0.500 \mathrm{~A} / \mathrm{s}$, the measured voltage is $\Delta V_{a b}=5.00 \mathrm{~V}$. Calculate the values of $L$ and $R$. (5 marks)
a)

b)

( $\left.10^{\text {pts }}\right)$ 14. Consider the mass spectrometer shown below. The magnitude of the electric field between the plates of the capacitor is $E=1000 \mathrm{~V} / \mathrm{m}$. A uniform magnetic field is directed into the page and has a magnitude of 0.1 T . Calculate the position $x$ at which a particle of mass $m=7.3 \times 10^{-26} \mathrm{~kg}$ and charge $q=1.6 \times 10^{-19} \mathrm{C}$ will hit the plate.

( $10^{\text {pts }}$ ) 15. A triangular circuit of total resistance $R$ is made from a conducting bar that can slide along a conducting track. A uniform magnetic field of magnitude $B$ is directed into the page as shown. If the bar is pulled to the right with speed $v$, find an expression for the induced current in terms of $B, v, x, \alpha$, and $R$. Is the induced current directed clockwise or counterclockwise?

( $\left.10^{\text {pts }}\right)$ 16. The toroid is a coil of wire wrapped around a doughnut-shaped ring made of a nonconducting material. Find an expression for the magnetic field strength at a point inside the coil of wire and a distance $r$ from the central axis.


Potentially Useful Formulae - page 1 of 2 Detach this sheet and keep it.

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\begin{aligned}
& g=9.81 \mathrm{~m} / \mathrm{s}^{2} \\
& K=8.99 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2} \\
& \text { Electron: } \quad q_{\mathrm{e}}=-e=-1.60 \times 10^{-19} \mathrm{C} \\
& v=v_{\mathrm{i}}+a_{\mathrm{c}} \Delta t \\
& x=x_{\mathrm{i}}+v_{\mathrm{i}} \Delta t+\frac{1}{2} a_{\mathrm{c}}(\Delta t)^{2} \\
& v^{2}=v_{\mathrm{i}}^{2}+2 a_{\mathrm{c}} \Delta x \\
& \vec{F}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} \hat{r} \\
& \Phi_{\mathrm{e}}=\int_{\text {surface }} \vec{E} \cdot d \vec{A} \\
& U_{\text {elec }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r} \\
& V=U_{\text {elec }} / q \quad V_{\text {net }}=\sum_{i} V_{i} \\
& \Delta V=V_{\mathrm{f}}-V_{\mathrm{i}}=-\int_{\mathrm{i}}^{\mathrm{f}} \vec{E} \cdot d \vec{s} \\
& E_{s}=-\frac{d V}{d s} \\
& C=\frac{Q}{\Delta V_{\mathrm{C}}} \\
& \text { parallel plate cap.: } C_{0}=\varepsilon_{0} \frac{A}{d} \quad C=\kappa C_{0} \\
& \text { parallel: } C_{\text {eq }}=\sum_{i} C_{i} \\
& \text { series: } \frac{1}{C_{\text {eq }}}=\sum_{i} \frac{1}{C_{i}} \\
& U_{\mathrm{C}}=\frac{Q^{2}}{2 C}=\frac{1}{2} C\left(\Delta V_{\mathrm{C}}\right)^{2}=\frac{1}{2} Q \Delta V_{\mathrm{C}} \\
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& \varepsilon_{0}=\frac{1}{4 \pi K}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2} \\
& m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg} \\
& \mu_{0}=4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}=1.257 \times 10^{-6} \mathrm{Tm} / \mathrm{A} \\
& \vec{A} \cdot \vec{B}=A B \cos \theta=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z} \\
& |\vec{A} \times \vec{B}|=A B \sin \theta \\
& \vec{E}=\vec{F} / q \quad \vec{E}_{\text {net }}=\sum_{i} \vec{E}_{i} \\
& \oint \vec{E} \cdot d \vec{A}=\frac{Q_{\text {in }}}{\varepsilon_{0}} \\
& V=U_{\text {elec }} / q \quad V_{\text {net }}=\sum_{i} V_{i} \\
& E_{s}=-\frac{d V}{d s}
\end{aligned}
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Potentially Useful Formulae - page 2 of 2 Detach this sheet and keep it.

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\begin{aligned}
& I=\frac{d Q}{d t} \\
& \sum I_{\mathrm{in}}=\sum I_{\mathrm{out}} \\
& R=\rho \frac{L}{A} \\
& \text { series: } R_{\mathrm{eq}}=\sum_{i} R_{i} \\
& P_{\text {bat }}=I \mathscr{E} \\
& \oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\mathrm{through}} \\
& \vec{F}=q \vec{v} \times \vec{B} \\
& \mathscr{E}=v \ell B \\
& \mathscr{E}=\left|\frac{d \Phi_{\mathrm{m}}}{d t}\right| \\
& \Delta V_{\mathrm{L}}=-L \frac{d I}{d t} \\
& J=\frac{I}{A}=n_{\mathrm{e}} e v_{\mathrm{d}} \\
& \Delta V_{\text {loop }}=\sum_{i}(\Delta V)_{i}=0 \\
& \Delta V_{\mathrm{R}}=I R \\
& \text { parallel: } \frac{1}{R_{\mathrm{eq}}}=\sum_{i} \frac{1}{R_{i}} \\
& P_{\mathrm{R}}=I \Delta V_{\mathrm{R}}=I^{2} R=\frac{\left(\Delta V_{\mathrm{R}}\right)^{2}}{R} \\
& B_{\text {loop }}=\frac{\mu_{0} I}{2 R} \\
& B_{\text {wire }}=\frac{\mu_{0} I}{2 \pi d} \\
& \vec{F}=I \vec{\ell} \times \vec{B} \\
& \Phi_{\mathrm{m}}=\int_{\text {area of loop }} \vec{B} \cdot d \vec{A} \\
& L=\frac{\Phi_{\mathrm{m}}}{I} \\
& U_{\mathrm{L}}=\frac{1}{2} L I^{2}
\end{aligned}
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